J. Fluid Mech. (1996), vol. 314, pp. 331–347

Internal hydraulic jumps at T-junctions

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(Received 15 October 1993 and in revised form 5 December 1995)

This paper presents a theoretical investigation of the occurrence of hydraulic jumps in two-layer systems induced by extraction of fluid from the upper layer. The physical configuration consists of a horizontal main pipe along which air and water flow, and a vertically upward side arm. An hydraulic model based on the momentum principle assuming that the fluids do not mix is developed that leads to at least two possible conjugate states for any given two-layer flow. A method of determining the amount of gas which must be extracted into the side arm for a jump to occur is developed and predictions shown to be in reasonable agreement with observation. Unusually, it is shown that above this critical gas take-off value two possible states remain energetically feasible.

1. Introduction

The division of a two-layer flow at a T-junction is important in the design of equipment used in oil and gas production and is a common feature of many chemical plants. Maldistribution of the fluids between the outlets can have a significant effect on the behaviour of equipment downstream of the junction far exceeding the size of the junction relative to the complete plant. For example, when steam injection is being used to effect enhanced recovery of viscous oils, the steam is usually generated at a central point and distributed to a number of wells. This can involve several junctions. In this process it is important to know where the water (either that coming from the boiler because of incomplete evaporation or that due to condensation of steam along the transmission lines) goes to, as water having much lower enthalpy than steam is much less effective at lowering the viscosity of the oil. Other examples include the pipework feeding a bank of air-cooled heat exchangers in the process industries, and loss of coolant accidents (LOCA) in nuclear power reactors of the Pressurized Water Reactor type. Apart from these cases where the phase separation can constitute a major problem, there are examples where the phenomenon has been used to advantage. In multi-bottle slug catchers used in oil and gas production, a two-phase flow is divided through a series of T-junctions into a number of large diameter pipes set at a small downward inclination. Each of these pipes contains a T-junction with a side arm emerging from the top of the main pipe. Most of the gas emerges from the side arm whilst the liquid continues along the main pipe. All gas outlets are manifolded to lead into the gas processing equipment, and there is a similar arrangement for the liquid. Obviously, correct operation relies on complete phase separation at the vertical side arm junctions. As discussed by Azzopardi (1993), the incorporation of T-junctions as partial separators into phase separation systems

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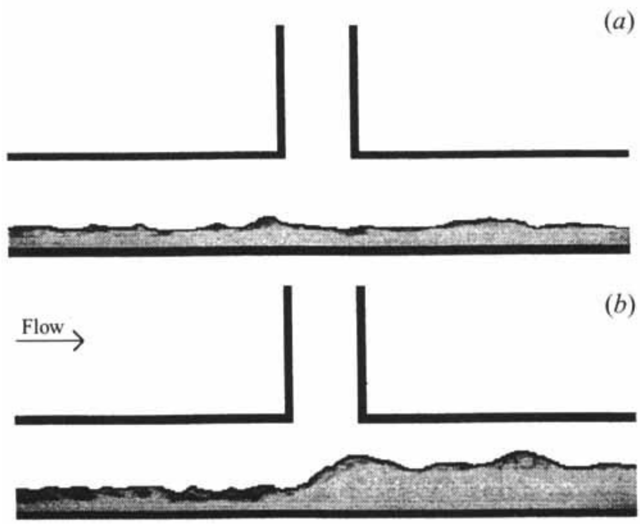

FIGURE 1. Example of the increase in the liquid level. (*a*) $G' \approx 0.55$ and (*b*) $G' \approx 0.75$. Gas flow rate $= 0.024$ kg s^{-1}, liquid flow rate $= 0.063$ kg s^{-1}, main tube diameter $= 0.038$ m, side arm diameter $= 0.025$ m, inlet pressure $= 3$ bar.

can lessen the load on the main separator, leading to smaller units which are easier to manufacture.

The split of an air–water flow at a T-junction with a horizontal main pipe of diameter 0.038 m and a vertically upward side arm of diameter 0.025 m has been studied experimentally by Azzopardi & Smith (1992). Under certain conditions they observed an abrupt change in the interface level of an approaching two-layer flow analogous to a hydraulic jump found in free surface flows. This phenomenon is shown illustratively in figure 1. A measure of the amount of gas extracted into the side arm is usually expressed as the mass fraction of the inlet gas flow which is taken off and is denoted by G' in this paper. There exists a critical value of gas take-off above which a significant increase in the liquid level occurs, and for the inlet conditions of figure 1, this value was observed by Azzopardi & Smith (1992) to be 0.69. At higher take-off values the increase in the liquid height and the presence of a local turbulence associated with the hydraulic jump facilitates the inception of liquid take-off. This leads to, for example, inefficient phase separation if the junction is being used as a phase separator, and hence prediction of the occurrence of hydraulic jumps is extremely important.

Experimental and theoretical studies of hydraulic jumps in one- and two-layer systems have been presented in the literature (see McCorquodale 1986). The number of works for the two-layer case is considerably less than that for a single-layer, and for jumps with negligible mixing of the layers, there are three main approaches which have been reported. The application of the momentum principle to individual layers was first suggested by Yih & Guha (1955) and use of this theory is still common (Rajaratnam, Tovell & Loewen, 1991). They considered flows in horizontal rectangular channels with a free upper surface and assumed that no momentum was transferred between the layers, the shear at the boundaries and the interface were negligible and a hydrostatic distribution of pressure. To enable separate momentum equations to be written for each layer, they made the additional assumption that the mean pressure over the jump section could be taken as the average of the upstream and downstream pressures at the interface. The solution of the resulting momentum equations together with the continuity equations for each layer yielded at most four possible conjugate depths including the upstream state itself. The determination of a unique conjugate state was investigated by Hayakawa (1970) on imposing the condition that energy loss is required at the hydraulic jump. It was found that there always existed a solution of the momentum equations which was not a legitimate solution. Mehrohtra & Kelly (1973) extended this approach to bounded two-layer flows, and suggested that of the two resulting legitimate solutions, the physical solution would approach an infinitesimally weak jump in the limit when the conditions upstream of the jump tend to the critical state. A conclusion was that the conjugate state that is closer to the upstream state was physically realizable for both open and closed channels.

Chu & Baddour (1977) and Wood & Simpson (1984) have queried the assumptions of Yih & Guha (1955) since they imply that the contracting layer gains energy without associated work being done on it. Chu & Baddour (1977) instead assumed that the energy loss in the contracting layer was negligible and that the combined momentum of the layers was conserved. Elimination of the unknown downstream pressure resulted in a particularly simple conservation relationship if prior knowledge of the contracting layer was available.

The theories of Yih & Guha (1955) and Chu & Baddour (1977) have been evaluated experimentally for jumps in the lee of a towed obstacle and for a jump advancing into stationary layers by Wood & Simpson (1984). Even though experiments were carried out with saline and fresh water layers and so jump phenomena would almost certainly involve an interfacial mixed layer, both theories whilst neglecting the mixing of the fluids gave similar results differing only when the shear on the interface became large.

A different approach has been presented by Armi (1986) for the special case of weak internal hydraulic jumps based on energy equations. If no energy is lost at the jump section, the resulting conservation-of-energy equations can be solved together with the continuity equations for a unique conjugate depth. This approach thus avoids the non-uniqueness problems inherent in the use of the momentum equations, but can exclude cases of major interest.

For internal hydraulic jumps at T-junctions, it is not obvious which of these theories, if any, is applicable to determine the liquid height downstream of the side arm. The situation is considerably more complex than for two-layer flow in a channel, and there are now two unknown pressure drops: one between the inlet and the side arm outlet, $\Delta P_{13}(= P_3 - P_1)$, and the other between the inlet and the downstream outlet in the main pipe, $\Delta P_{12}(= P_2 - P_1)$. Subscripts 1 and 2 are used to indicate upstream and downstream sections in the main pipe, and 3 the flow in the side arm. The fraction of gas taken off into the side arm, G', is effectively a measure of ΔP_{13}, leaving the pressure drop ΔP_{12} as an unknown parameter. In §2, we argue that ΔP_{12} must be given for this system by determining the direction information is propagating in the downstream flow.

Although there may be a loss of energy between the upstream and downstream states of a jump due to turbulence, momentum is always conserved. In §3, the momentum principle is applied to the tee and the resulting equation solved to yield the possible downstream liquid heights for given values of ΔP_{12} and G'. The results are compared in §4 with measurements of the liquid height reported by Azzopardi & Smith (1992) for which G', but not ΔP_{12}, is known.

It is debatable whether there is a sufficient loss of energy at the jump to discount a theory utilizing an energy equation for the contracting layer (Chu & Baddour

1977), or for both layers (Armi 1986). Either theory allows the downstream liquid layer height to be determined for a given gas take-off value without prescribing ΔP_{12}, contradicting the arguments presented in § 2. Nonetheless, for completeness, the results of the extension of the theories of Chu & Baddour (1977) and Armi (1986) to account for fluid extraction from the upper layer are presented in § 5, and results shown to be in poor agreement with data. Finally, in § 6 we suggest a method to determine the critical gas take-off, and present conclusions in § 7.

2. Information propagation

It is important to be aware of which direction information is propagated by long, small-amplitude gravity waves on the interface between the fluids in the main pipe of the T-junction. By analysing such waves in a channel of uniform rectangular cross-section with arbitrary velocities in the two layers, Armi (1986) showed that the two characteristic velocities are given by

$$\lambda^{\pm} = u_{con} \pm c, \tag{2.1}$$

where the convective velocity, u_{con}, and the phase speed, c, are given by

$$u_{con} = \frac{r u_G h_L + u_L h_G}{h_G + h_L}, \tag{2.2}$$

and

$$c = \left\{ g' \frac{h_G h_L}{h_G + h_L} \left[1 - \frac{r(u_G - u_L)^2}{g'(h_G + h_L)} \right] \right\}^{1/2}, \tag{2.3}$$

where u, h, ρ are the layer velocity, thickness and density respectively, subscripts G and L denote the gas and liquid phases, $r = \rho_G/\rho_L$ and $g' = (1-r)g$. As explained by Dalziel (1991), if the characteristic velocities are of opposite signs, information is able to propagate in both directions and the flow is said to be subcritical. In contrast, if the velocities are of opposite sign, information about any disturbance is able to propagate in one direction only and the flow is said to be supercritical. The ratio of the convective velocity, u_{con}, to the phase speed, c, is traditionally known as the Froude number, Fr, see Lawrence (1990). The flow is thus supercritical or subcritical depending on whether the Froude number is greater than or less than unity.

Azzopardi & Smith (1992) measured the downstream liquid height at various gas take-off values with inlet gas and liquid mass flow rates of 0.024 and 0.063 kg s^{-1} respectively. To calculate the characteristic velocity or Froude number of these flows, one has to account for the difference in the geometry in which the measurements were taken and the above equations apply. As suggested by Chow (1959), the liquid depth used in the channel flow calculations is assumed to be equal to a liquid depth defined as the area occupied by the liquid divided by the width of the interface. It is also assumed that the channel has a square cross-section of width equal to the pipe diameter, and the upstream velocities determined from the inlet mass flow rates. To calculate the downstream velocities for a given gas take-off, the continuity equations (3.1a) and (3.1b) are applied. The resulting characteristic velocities and Froude numbers for the downstream heights measured by Azzopardi & Smith (1992) dimensionalized with respect to the pipe diameter D are shown in table 1. The downstream flow is calculated to be supercritical for gas take-off values below 0.622, and subcritical above 0.715 when a significant increase in the downstream liquid height occurs. We thus argue that the downstream pressure, or the pressure change ΔP_{12},

G'	h/D	λ^+	λ^-	Fr
0.0	0.17	0.493	0.145	1.831
0.622	0.22	0.520	0.089	1.408
0.715	0.38	0.532	−0.011	0.960
0.798	0.39	0.533	−0.019	0.933
0.913	0.39	0.533	−0.021	0.926
0.958	0.38	0.533	−0.015	0.944

TABLE 1. Calculated values of the characteristic velocities and Froude number

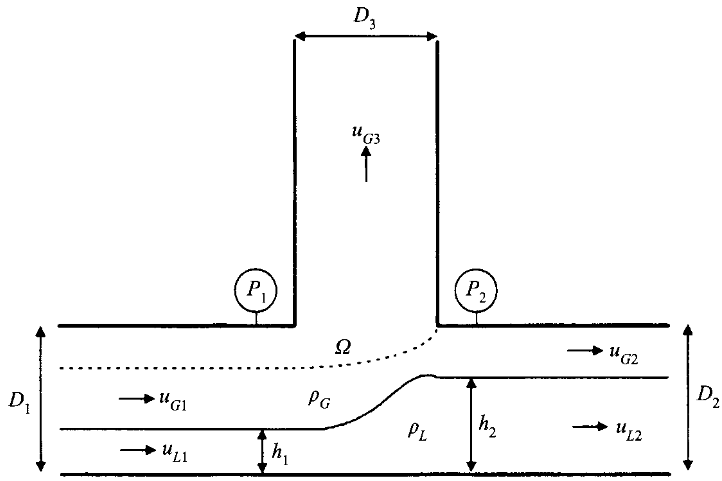

FIGURE 2. A definition sketch of the flow at the T-junction.

must be specified for this problem to determine the downstream liquid height when a significant increase occurs and the flow is subcritical with information propagating in both directions.

Further evidence is provided by observation of the waves on the common interface between the two layers. Energy loss from a mean flow can, and often does, manifest itself in wave energy (Benjamin & Lighthill 1954). Video recordings of the interface reveal that a stationary wave train exists in the downstream arm which may carry energy away from the junction. This would indicate that the flow is subcritical and influenced by the downstream boundary conditions, as calculated above.

3. The model

A definition sketch is shown in figure 2 depicting a T-junction with a horizontal main pipe and a vertically upward side arm. The pressure at the upper bounding surfaces, the mean velocity, the density, the phase area and the liquid height are denoted by P, u, ρ, A and h respectively. Subscripts 1 and 2 are used to indicate upstream and downstream sections in the main pipe, and 3 the flows in the side arm.

For the purposes of this section, we shall determine the downstream liquid height given the gas take-off for various values of the pressure change along the main pipe ΔP_{12}. The usual three assumptions used in the study of two-layer flow known as the hydraulic assumptions are applied: (i) the fluids are inviscid, (ii) the pressure is hydrostatic, and (iii) within each layer the density is constant and the velocity only varies in the flow direction. Energy loss at the jump, however, is assumed to occur by viscous dissipation. It is also assumed that there is no mixing of the fluids.

3.1. *Continuity equations*

If liquid is not extracted into the side arm, the gas and liquid continuity equations can be written as

$$q_{G1}(1-G') = q_{G2}, \tag{3.1a}$$

and

$$q_{L1} = q_{L2}, \tag{3.1b}$$

where $q_{Ki}(= u_{Ki}A_{Ki})$ is the volumetric flow rate of phase K at section i, and G' is the mass fraction of inlet gas flow which is extracted into the side arm.

3.2. *The momentum equation*

An equation relating conditions across the jump may be obtained from conservation of total momentum expressed as the flow force S (the horizontal pressure force due to depth changes plus momentum flow rate over a cross-section), see Baines (1984) and Wood & Simpson (1984). The flow force of the upper and lower layers upstream and downstream of the side arm depends upon the cross-sectional shape of the channel and expressions are given in Appendix A for channels of square and circular cross-section.

There is a contribution to the net horizontal momentum due to the side arm since the action of the turning fluid and the presence of a recirculation zone in the side arm results in the normal pressure along the boundary on the inlet side being less than that on the opposite side. A net horizontal boundary force, termed the junction force F_x, thus acts in the inlet direction. A correlation for the junction force with only gas flowing through the tee has been given by Katz (1967), who assumed that the junction force was proportional to the momentum flux of the turning fluid. The constant of proportionality, the so-called Katz parameter, was found to be 0.7 by measuring the pressures at the upstream and downstream sections. This approach has been successfully extended to two-phase flow by Davis & Fungtamasan (1990) in their investigation of the flow of a gas–liquid mixture in the froth–bubbly flow regime at a T-junction with a vertical main pipe. They related the junction force to the inlet flow mass flux density and branch flow velocity as follows:

$$F_x = k_x[\alpha_1\rho_G u_{G1}u_{G3} + (1-\alpha_1)\rho_L u_{L1}u_{L3}]A_3, \tag{3.2}$$

where α_1 is the upstream void fraction of gas and A_3 is the cross-sectional area of the side arm. The Katz parameter was found experimentally to have roughly the same value as in single-phase flow and hence is taken to be the widely used value of 0.7 in the subsequent calculations (§4). If liquid is not extracted into the side arm, the junction force upon non-dimensionalization with respect to $\rho_L g D^3$ becomes

$$F_x^* = rk_xG'\frac{q_{G1}^{*\,2}}{A_1^*}, \tag{3.3}$$

where $A^* = A/D^2$ and $q^* = q/(g^{1/2}D^{5/2})$.

Conservation of the horizontal momentum results in

$$S_{G1} + S_{L1} = S_{G2} + S_{L2} + F_x, \tag{3.4}$$

assuming that the jump is short enough for the shear at the boundaries to be negligible.

For a T-junction made up of channels of square cross-section, algebraic manipulation of the above non-dimensionalized equation using the expressions for the flow forces given in Appendix A then yields, upon elimination of the downstream flow rates, the following equation for the dimensionless downstream liquid height $d_2 = h_2/D$:

$$
\begin{aligned}
K_S(d_2; d_1, q_{G1}, q_{L1}, r, G', \Delta P) \equiv & -\tfrac{1}{2}sd_2^4 + \tfrac{1}{2}sd_2^3 \\
& + \left[\tfrac{1}{2}sd_1^2 - \Delta P + \frac{rq_{G1}^2}{1-d_1} + \frac{q_{L1}^2}{d_1} - F_x\right] d_2^2 \\
& + \left[-\tfrac{1}{2}sd_1^2 + \Delta P + rq_{G1}^2(1-G')^2 - q_{L1}^2 - \frac{rq_{G1}^2}{1-d_1} - \frac{q_{L1}^2}{d_1} + F_x\right] d_2 \\
& + q_{L1}^2 = 0,
\end{aligned} \tag{3.5a}
$$

where the * notation is now dropped for convenience, $d_1 = h_1/D, s = 1-r, \Delta P = P_{12}(= P_2 - P_1)$ and as in most practical situations $D_1 = D_2 = D$. For a T-junction made up of channels of circular cross-section, the conservation of the total momentum yields

$$
\begin{aligned}
K_C(\theta_2; \theta_1, q_{G1}, q_{L1}, r, G', \Delta P) \equiv & \frac{\pi}{4}\Delta P + \frac{q_{L1}^2}{A_{L1}}\left[\frac{A_{L1}}{A_{L2}} - 1\right] + \frac{rq_{G1}^2}{A_{G1}}\left[\frac{A_{L1}}{A_{L2}}(1-G')^2 - 1\right] \\
& + \tfrac{1}{2}s(\theta_2 A_{L2} - \theta_1 A_{L1}) + \tfrac{1}{12}s[(1-\theta_2^2)^{3/2} - (1-\theta_1^2)^{3/2}] \\
& + F_x = 0,
\end{aligned} \tag{3.5b}
$$

where $\theta = 2d - 1$.

3.3. *Energy loss*

The main energy losses in the jump region are due to the recirculation zone which exists in the side arm and turbulence. We assume that there exists a dividing stream surface Ω which separates the inlet gas flow which is extracted into the side arm from the fluids which continue to flow downstream along the main pipe as shown in figure 2. If the energy exchange through Ω is negligible, the energy losses can be considered separately. The total loss in the energy flux, ΔE, due to the change in the downstream liquid height is thus taken to be the loss in the energy flux of the fluids which flow downstream in the main pipe. Denoting the specific energy in any layer by E with the proper suffix

$$\Delta E = E_{G1}u_{G1}A_{G1}(1-G') + E_{L1}u_{L1}A_{L1} - E_{G2}u_{G2}A_{G2} - E_{L2}u_{L2}A_{L2}. \tag{3.6}$$

Non-dimensionalizing with respect to $\rho_L g^{3/2} D^{7/2}$ and using the continuity relations, the energy loss can be written as

$$\Delta E^* = (E_{G1}^* - E_{G2}^*)q_{G1}^*(1-G') + (E_{L1}^* - E_{L2}^*)q_{L1}^*, \tag{3.7}$$

where the dimensionless specific energies are

$$E_G^* = P^* + r + \frac{rq_G^{*2}}{2A_G^{*2}}, \tag{3.8a}$$

and

$$E_L^* = P^* + r + (1-r)d + \frac{q_L^{*2}}{2A_L^{*2}}. \tag{3.8b}$$

The dimensionless energy loss of a jump is thus

$$\Delta E = \left\{ \frac{r q_{G1}^2}{2A_{G1}^2} \left[1 - \left(\frac{A_{G1}}{A_{G2}} \right)^2 (1 - G')^2 \right] - \Delta P \right\} q_{G1}(1 - G') + \left\{ \frac{q_{L1}^2}{2A_{L1}^2} \left[1 - \left(\frac{A_{L1}}{A_{L2}} \right)^2 \right] + s(d_1 - d_2) - \Delta P \right\} q_{L1}, \tag{3.9}$$

where the * notation has been dropped for convenience.

As take-off increases the portion of the inlet flow which continues downstream along the main pipe decreases and hence the flux of energy into the jump region decreases. The energy loss of a jump is thus displayed in the following sections as the relative energy loss, η, between the inlet and outlet sections where

$$\eta = \frac{\Delta E}{E_1}, \tag{3.10}$$

and the flux of energy into the system is

$$E_1 = E_{G1} q_{G1}(1 - G') + E_{L1} q_{L1}. \tag{3.11}$$

For a root of the momentum equation (3.5*a*) or (3.5*b*) to be acceptable, the resulting change in the interface height must be accompanied by a loss of energy (Hayakawa 1970). Legitimate solutions must therefore satisfy the following conditions:

$$0 \leqslant d_2 \leqslant 1, \tag{3.12a}$$

$$\eta \geqslant 0. \tag{3.12b}$$

4. Predictions

Solutions of the momentum equations (3.5*a*) and (3.5*b*) are sought at all take-off values for a range of pressure changes, ΔP, greater than zero. For a T-junction composed of channels of square cross-section, a map of the ratio of the downstream to the upstream liquid height versus the fraction of gas taken off is displayed in figure 3(*a*). There are two solutions satisfying (3.12*a*) for a range of take-off values provided that the pressure change along the main pipe is less than some critical value, otherwise no valid solutions occur. The relative energy loss of the solutions in figure 3(*a*) is shown in figure 3(*b*).

If there is no loss in the energy flux between the upstream and downstream sections, equation (3.9) can be manipulated to give an expression for the pressure change in the main pipe:

$$\Delta P = \frac{r q_{G1}^2}{2A_{G1}^2} \left[1 - \left(\frac{A_{G1}}{A_{G2}} \right)^2 (1 - G')^2 \right] \frac{q_{G1}(1 - G')}{q_{G1}(1 - G') + q_{L1}} + \left\{ \frac{q_{L1}^2}{2A_{L1}^2} \left[1 - \left(\frac{A_{L1}}{A_{L2}} \right)^2 \right] + s(d_1 - d_2) \right\} \frac{q_{L1}}{q_{G1}(1 - G') + q_{L1}}. \tag{4.1}$$

Upon elimination of the pressure change between equations (3.5*a*) and (4.1) and factorization of the full channel and zero liquid height solutions, we obtain the sixth-order polynomial given in Appendix B for the downstream liquid heights at which there is no loss in the energy flux. Physical solutions of this equation are represented

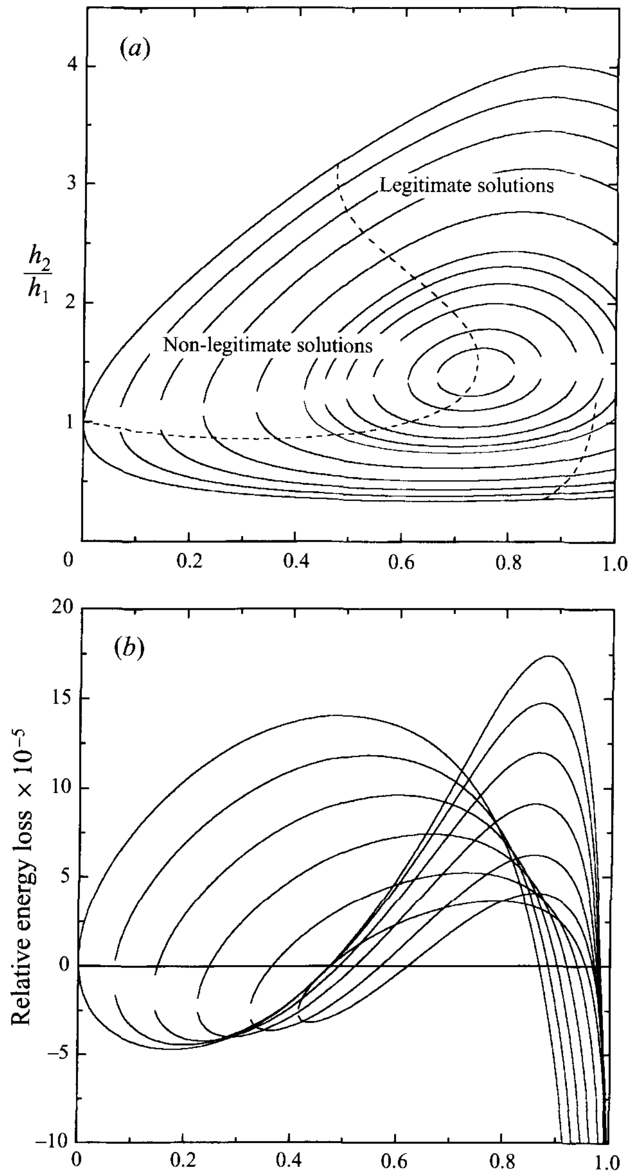

FIGURE 3. (*a*) Solutions to the momentum equation (3.5*a*) for a T-junction composed of channels of square cross-section. The largest to the smallest solution curves correspond to pressure changes of 0, 12, 24, 36, 48, 57, 60, 63, 66, 69 and 70.5 N m^{-2}. The dotted line (- - - -) corresponds to the conjugate states for which the loss of energy is zero. (*b*) The dimensionless energy loss curves. The curves which are the largest at low take-off values correspond to the first six lower solution curves shown in (*a*). Similarly for the other curves. Inlet conditions and pipe geometry are the same as in figure 1.

by the boundaries (the dotted lines) in figure 3(*a*). The solution plane is thus divided into regions of legitimate and non-legitimate solutions where the energy loss due to the change in the liquid level is either positive or negative respectively. It can be seen that for differing take-off values, there are either one or two legitimate conjugate states.

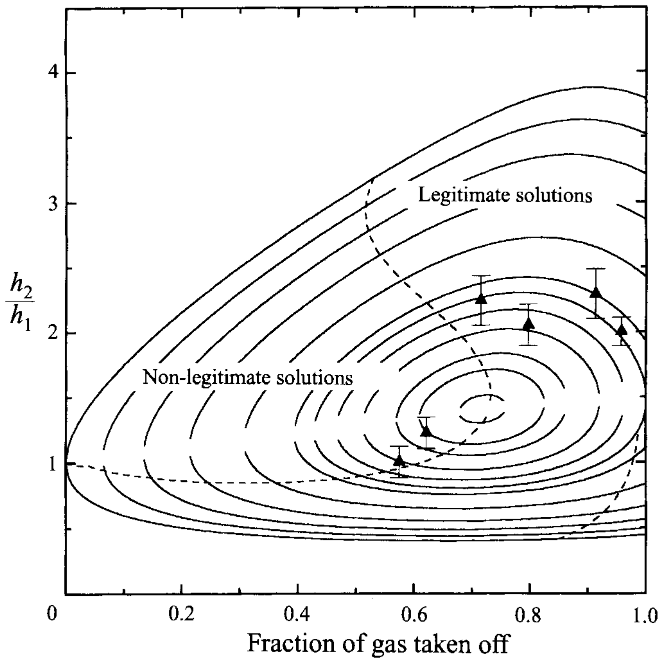

FIGURE 4. Solutions to the momentum equation (3.5*b*) for a T-junction composed of channels of circular cross-section. The largest to the smallest solution curves correspond to pressure changes of 0, 12, 24, 36, 48, 57, 60, 63, 66, 69, 70.5 and 72 N m^{-2}. The dotted line (- - - -) corresponds to the conjugate states for which the loss of energy is zero. The upward triangles represent experimental data from Azzopardi & Smith (1992). Inlet conditions and pipe geometry are as in figure 1.

The corresponding solution map for a T-junction composed of circular channels is displayed in figure 4. The energy boundary is obtained by iterative solution of the momentum equation (3.5*b*) with the pressure change given by equation (4.1). The region of legitimate solutions can be seen to be very similar to that calculated with channels of square cross-section. If the take-off value is less than about 0.52, only one legitimate conjugate state exists. If the take-off value is between 0.52 and 0.73 or higher than 0.85, one or two legitimate conjugate states can exist depending on the downstream pressure. For the remaining take-off values, both solutions are found to be energetically possible. The predictions are compared with the measurements of the downstream liquid height reported by Azzopardi & Smith (1992) for which G', but not ΔP, is known. It should be noted that these measurements were taken from stills of high-speed video recordings and so may involve significant errors as indicated. All but one of the data points lie in the legitimate region; the exceptional point corresponds to a particularly wavy interface and is close enough to the energy boundary so as not to present undue concern for the validity of the theory.

For take-off values above 0.715, the observed downstream liquid levels correspond to theoretical pressure changes ΔP in the range 57–66 N m^{-2}, or in dimensionless terms $\Delta P^*(=\Delta P/\rho_L g D)$, in the range 0.153–0.177. Azzopardi & Smith (1992) did not measure the downstream pressure and so direct comparison of results is not possible. However the pressure change along the main pipe of low-pressure (1.5 bar) two-layer flows at a T-junction composed of 0.038 m diameter pipes all on the same horizontal plane has recently been measured by Buell, Soliman & Sims (1993). The pressure change for three different inlet flows was found to be in the range 9–152 N m^{-2}

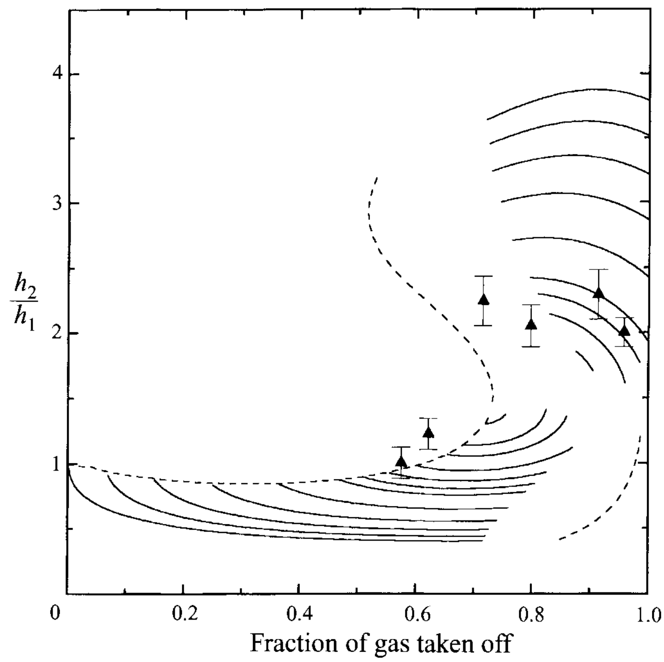

FIGURE 5. Solutions shown in figure 4 with the greatest energy loss.

with gas take-off values in the range 0.14–0.98. The magnitude of the theoretical pressure required to predict the observed jump heights is consistent with these typical measured values.

For pressure changes in the range 57–66 N m^{-2}, there also are legitimate solutions to the momentum equation which are closer to the initial state but are not observed. This is in direct contrast to the conclusions of Mehrotra & Kelly (1973). The maximum number of possible conjugate states of a given two-layer flow is, however, the same as predicted by Hayakawa (1970). The determination of the solution which actually occurs is a 'classical' problem inherent in the application of the momentum principle to which there is no obvious solution. If it is assumed that the state with the largest positive energy loss will be observed, then theory predicts the unique solutions shown in figure 5. Although a jump is correctly predicted to occur in preference to a drop in the interface level at high take-off values, only small increases or drops are found at take-off values of 0.715 and 0.798 at which jumps were observed for pressure changes in the range 57–66 N m^{-2}. Determination of a unique conjugate state via energy arguments thus yields poor agreement with observation.

5. Jump height predictions without prescribing ΔP_{12}

It may be possible that the energy losses that occur at the jump are not significant enough to discount a model based on the conservation of energy. If energy losses are negligible in the contracting (upper) layer (Chu & Baddour 1977), the pressure change along the main pipe can be obtained from the Bernoulli equation for the gas phase. It can be seen from equation (3.9) that the pressure change is then given by

$$\Delta P = \frac{rq_{G1}^2}{2A_{G1}^2}\left[1 - \left(\frac{A_{G1}}{A_{G2}}\right)^2 (1 - G')^2\right]. \tag{5.1}$$

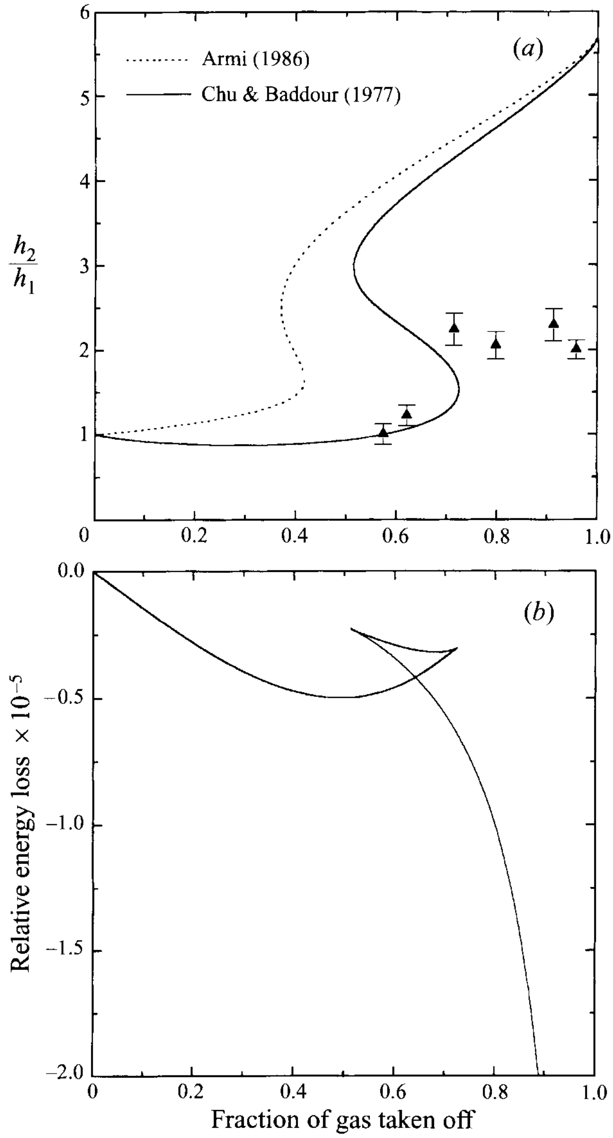

FIGURE 6. (*a*) Predictions obtained by applying the theories of Chu & Baddour (1977) and Armi (1984). The upward triangles represent experimental data from Azzopardi & Smith (1992). (*b*) The relative energy loss of the solutions predicted by applying the theory of Chu & Baddour (1977). Inlet conditions and pipe geometry as in figure 1.

Solutions of the momentum equation (3.5*b*) with the pressure change given by (5.1) are shown in figure 6(*a*) and the resulting relative energy loss of the liquid layer in figure 6(*b*). The predictions significantly overpredict the jump heights observed by Azzopardi & Smith (1992) and, moreover, there is always a gain in energy in the liquid layer instead of a loss, implying that energy is unphysically created within the system.

A simplified model based entirely on the conservation of energy can be formulated if it is assumed that the energy loss in both layers is negligible (Armi 1986). The pressure change along the main pipe can then be obtained from equation (5.1), or the

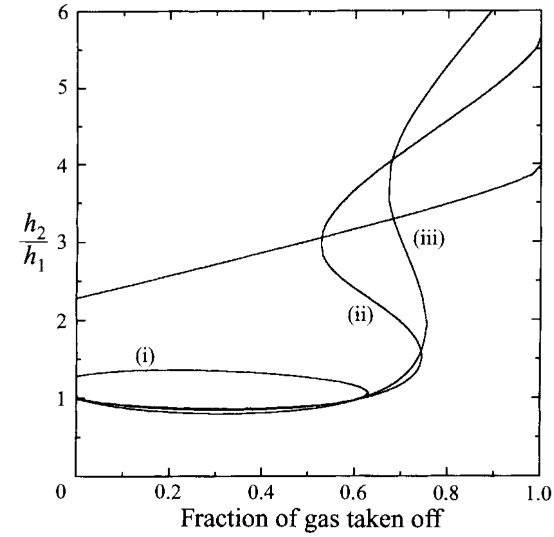

FIGURE 7. Energy boundaries for a T-junction composed of channels of circular cross-section. Liquid flow rate is 0.063 kg s^{-1}; gas flow rate is (i) 0.0126, (ii) 0.024 and (iii) 0.035 kg s^{-1}. Pipe geometry as in figure 1.

following energy equation for the liquid:

$$\Delta P = s(d_1 - d_2) + \frac{q_{L1}^2}{2A_{L1}^2}\left[1 - \left(\frac{A_{L1}}{A_{L2}}\right)^2\right]. \tag{5.2}$$

Subtraction of equation (5.1) from (5.2) removes the dominant effect of hydrostatic pressure not associated with the internal dynamics and the following expression is derived for the downstream liquid height:

$$K_C(d_2; d_1, q_{G1}, q_{L1}, r, G') \equiv s(d_1 - d_2) + \frac{q_{L1}^2}{2A_{L1}^2}\left[1 - \left(\frac{A_{L1}}{A_{L2}}\right)^2\right] - \frac{rq_{G1}^2}{2A_{G1}^2}\left[1 - \left(\frac{A_{G1}}{A_{G2}}\right)^2 (1-G')^2\right] = 0. \tag{5.3}$$

An advantage of this method is that a correlation for the junction force is not required. However, the solutions of equation (5.3) are shown in figure 6(a) to significantly overpredict the data of Azzopardi & Smith (1992).

6. Critical gas take-off

In this section, we propose a method to determine the critical gas take-off value above which hydraulic jumps and the associated problems may occur based on the model of §2. The energy boundary curves obtained by solution of the momentum equation (3.5b) with the pressure change given by equation (4.1) are shown in figure 7 for three different inlet gas flow rates for which measurements of the critical gas take-off are reported by Azzopardi & Smith (1992). As the gas take-off is increased, the lower solutions for each flow rate represent only slight changes in the liquid level until the curve doubles back on itself, after which only large solutions exist with further increases in gas take-off. These predicted liquid heights represent extremely large

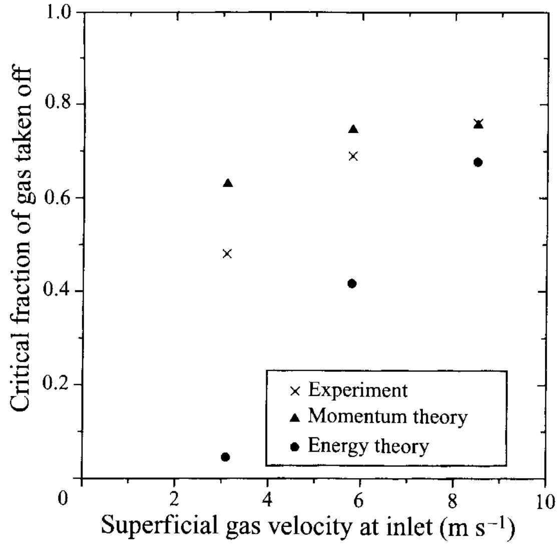

FIGURE 8. A comparison between theoretical values of the critical gas take-off and the values observed by Azzopardi & Smith (1992).

jumps and there would consequently be a great deal of energy loss in both phases which contradicts the initial assumption made to calculate the solutions. The gas take-off value at which the turning point of each curve occurs is thus the maximum value beyond which a significant increase in the liquid level can be expected and is thus taken to be the critical gas take-off value at which the jump will form. Figure 8 compares the resulting critical values with those reported by Azzopardi & Smith (1992). Good agreement is found for the highest flow rate cases with deviation from the data increasing as the flow rate decreases. There is no obvious reason why this trend should be obtained and it may not be observed if more data were available for comparison with predictions of the model.

Applying a similar argument to the predictions obtained from solution of equation (5.3) based entirely on an energy approach results in critical take-off values which are significantly lower than the observed values as shown in figure 8.

7. Conclusions

A model has been derived based on the conservation of momentum to determine the liquid height downstream of a vertically upward side arm of a T-junction given the inlet flow conditions, the gas take-off and the pressure change along the main pipe. Jump heights are predicted which are in agreement with observation; however a drop solution is also energetically possible for the same gas take-off value and pressure change. The determination of at least two downstream states to a given flow is a 'classical' result inherent in the use of the momentum principle to which there is no obvious solution.

Theories utilizing the conservation of energy in either the contracting layer or both layers allow the downstream liquid height to be determined without prescription of the pressure change along the main pipe. However, results are in very poor agreement with observation. This gives evidence to support the argument that the pressure change along the main pipe must be specified and cannot be predicted when a jump occurs due to the flow being subcritical with information propagating in both directions.

Although the model cannot predict the jump height unless the pressure change is known and a method of resolving the non-uniqueness problem is found, the flow conditions at which jumps and consequent problems in a system incorporating a tee may occur can be predicted. By assuming that there is no energy flux loss between the upstream and downstream sections of the jump until a significant increase in the downstream liquid height occurs, the critical gas take-off value is predicted in reasonable agreement with observation.

The tentative conclusions presented in this paper are based on a limited amount of data, and the analysis is hoped to prompt the collection of further experimental measurements to thoroughly test the validity of the results and to resolve the non-uniqueness problem.

P. A. Roberts was supported by a CASE award from the Science and Engineering Research Council and AEA Technology. The authors would like to thank Professor B. J. Azzopardi for bringing this problem to our attention and acknowledge numerous helpful discussions. The comments of the referees in the preparation of this revised version are also gratefully acknowledged, particularly those contributed by Professor D. H. Peregrine.

Appendix A. The flow force expressions

The flow forces of the gas and liquid are

$$S_G = \int_{A_G} [P + \rho_G g(D-z) + \rho_G u_G^2] \mathrm{d}A(z), \tag{A 1a}$$

$$S_L = \int_{A_L} [P + \rho_G g(D-h) + \rho_L g(h-z) + \rho_L u_L^2] \mathrm{d}A(z), \tag{A 1b}$$

where A_k is the area occupied by phase k. Upon defining the dimensionless depth of the centre of gravity below the interface to be

$$\bar{d} = \frac{1}{A_L D} \int_0^h (h-z) \mathrm{d}A(z), \tag{A 2}$$

the following expressions are obtained:

$$S_G^* = r\left\{ \left[\frac{P^*}{r} + 1 - d + \left(\frac{q_G^*}{A_G^*} \right)^2 \right] A_G^* + \left(d - \frac{1}{2} \right) A^* - \bar{d} A_L^* \right\}, \tag{A 3a}$$

$$S_L* = \left[P^* + r(1-d) + \bar{d} + \left(\frac{q_L^*}{A_L^*} \right)^2 \right] A_L^*. \tag{A 3b}$$

For flows in a channel of square cross-section:

$$\bar{d} = \tfrac{1}{2} d; \tag{A 4}$$

$$A_L^* = d; \tag{A 5}$$

$$A_G^* = 1 - d, \tag{A 6}$$

and for pipe flows:

$$\bar{d} = \frac{1}{12A_L^*}(1-\theta^2)^{3/2} + \tfrac{1}{2}\theta; \tag{A 7}$$

$$A_L^* = \tfrac{1}{4}[\pi - \cos^{-1}\theta + \theta(1-\theta^2)^{1/2}]; \tag{A 8}$$

$$A_G^* = \tfrac{1}{4}\pi - A_L^*, \tag{A 9}$$

where $\theta = 2d - 1$.

Appendix B. The energy boundary equation for channels of square cross-section

$$K_s(d_2; d_1, q_{G1}, q_{L1}, r, G') \equiv C_6 d_2^6 + C_5 d_2^5 + C_4 d_2^4 + C_3 d_2^3 + C_2 d_2^2 + C_1 d_2 + C_0 = 0, \tag{B 1}$$

where

$$C_6 = \tfrac{1}{2}s; \quad C_5 = -s\left(1 + \frac{q_{L1}}{Q}\right);$$

$$C_4 = \tfrac{1}{2}s(1-d_1)^2 - \frac{rq_{G1}^2}{1-d_1} - \frac{q_{L1}^2}{d_1} + \frac{rq_{G1}^3(1-G')}{2(1-d_1)^2 Q} + \frac{q_{L1}^3}{2d_1^2 Q} + \frac{sq_{L1}(2+d_1)}{Q} + F_x;$$

$$C_3 = sd_1^2 + \frac{2rq_{G1}^2}{1-d_1} + \frac{2q_{L1}^2}{d_1} - rq_{G1}^2(1-G')^2 + q_{L1}^2 - \frac{rq_{G1}^3(1-G')}{(1-d_1)^2 Q} - \frac{q_{L1}^3}{d_1^2 Q} - \frac{sq_{L1}(1+2d_1)}{Q} - 2F_x;$$

$$C_2 = -\frac{1}{2}sd_1^2 - \frac{rq_{G1}^2}{1-d_1} - \frac{q_{L1}^2}{d_1} + rq_{G1}^2(1-G')^2 - 2q_{L1}^2 + \frac{rq_{G1}^3(1-G')}{2(1-d_1)^2 Q}[1-(1-d_1)^2(1-G')^2] + \frac{q_{L1}^3}{2d_1^2 Q}(1-d_1^2) + \frac{sq_{L1}d_1}{Q} + F_x;$$

$$C_1 = q_{L1}^2\left(1 + \frac{q_{L1}}{Q}\right); \quad C_0 = -\frac{q_{L1}^3}{2Q},$$

and

$$Q = q_{G1}(1-G') + q_{L1}.$$

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